II. EVALUATION OF RESPONSE

A. GEOMETRY OF A SUBMARINE

A typical submarine hull is considered as a generic body of revolution, which is rotated about a line parallel to the center-line, as described by Jackson (1992) [Ref. 3]. It has a length/diameter (L/D) ratio of six and a maximum diameter at $0.4L$. The body is composed of three main sections, as shown in Figure 1 [Ref. 3]. The forward section is called the entrance, which is a portion of an ellipsoid of revolution. The middle section is the parallel middle body (PMB) with a cylindrical shape. The third section is the after end called the run, which is composed of a paraboloid of revolution. The entrance has a length, L_f , of 2.4 diameters. The run has a length, L_a , of 3.6 diameters. The algebraic sum of the lengths, L_f , L_a , and the length of the PMB , L_{PMB} , is the overall length of the hull.

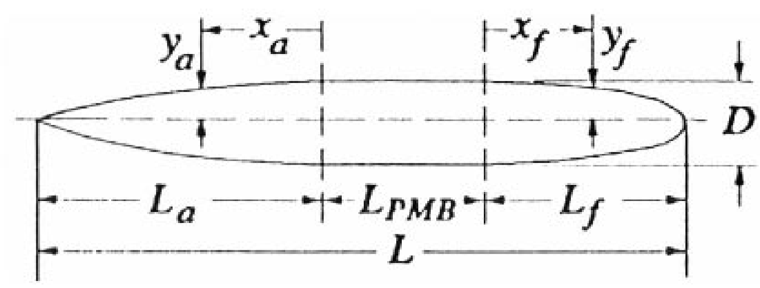

Figure 1. Submarine Geometry [Ref. 3]

The body coordinates, which define the forward and aft shapes are given by,

$$y_f = \frac{D}{2}\left[1-\left(\frac{x_f}{L_f}\right)^{n_f}\right]^{1/n_f} \tag{1}$$

$$y_a = \frac{D}{2}\left[1-\left(\frac{x_a}{L_a}\right)^{n_a}\right] \tag{2}$$

The values of x_f and x_a are the offsets from the maximum diameter, and y_f and y_a are the radii at the respective offset points.

The exponents (n_a and n_f) in Equations (1) and (2) are the shape factor coefficients, which control the shape of the fore and aft bodies, respectively. Higher values of these coefficients correspond to fuller hull shapes, and lower values to finer shapes. The effects of changing the shape factor coefficients on the hull shape are shown in Figures 2 and 3, where the hull shapes for three values of the shape factors 2, 3 and 4 are shown. In this study, it is assumed that the total volume of the ship remains the same (so that ship displacement does not vary), while either overall length or diameter remains the same. We refer to the first case as the limited length case or inactive diameter constraint, and to the second case as the limited diameter case or inactive length constraint. Both of these two cases will be analyzed in our parametric studies.

If one were to use equations for true ellipsoids and parabolas, the entrance and the run would be too fine for a modern submarine. The displacement can be increased by using larger shape factors or higher L_{PMB}. The prismatic coefficients , C_{pa} and C_{pf} ,are used to calculate volumes. For a cylinder its prismatic coefficient is 1. For a submarine-like

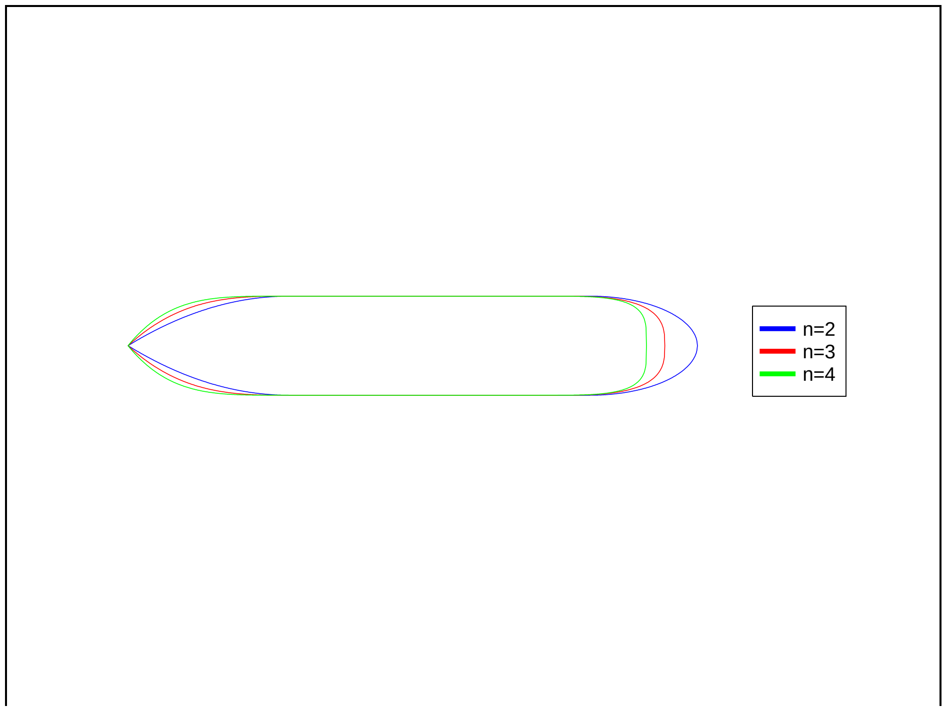

Figure 2. The effect of changing the shape factors for the limited diameter case.

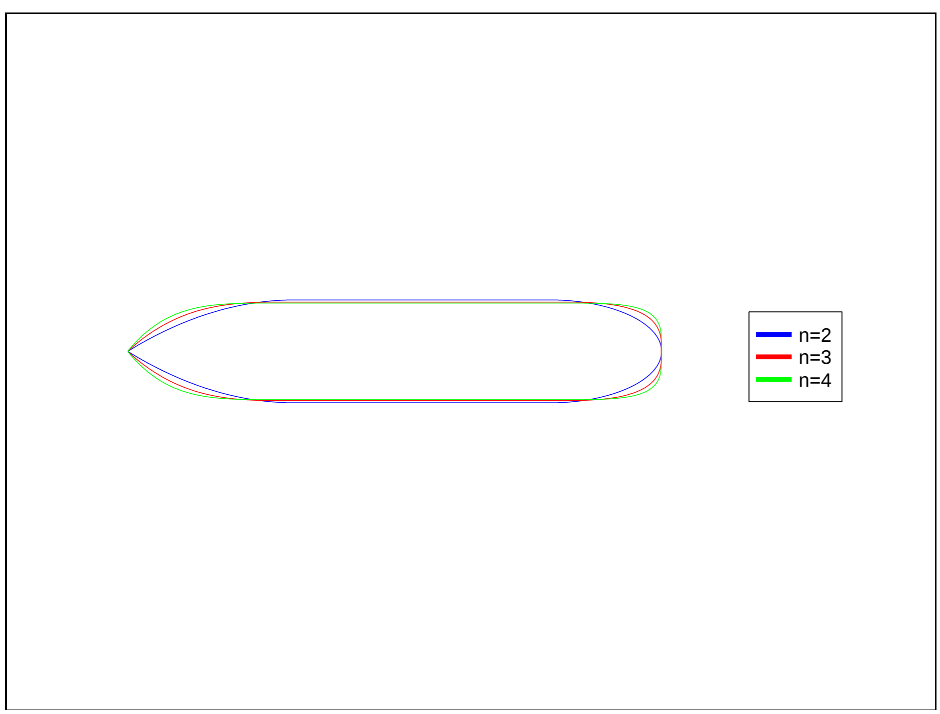

Figure 3. The effect of changing the shape factors for the limited length case.

body the prismatic coefficient can be evaluated in terms of its geometry. Using the above concept, L_{PMB} is the difference between the overall length and six times the diameter, that is *L-6D*. A method of calculating the volume of the entire hull can be developed by calculating the volume of each section separately by using the expressions above. Let V_f, V_a, and V_{PMB} denote volumes of the entrance, the run, and the PMB, respectively. The resulting equations are:

$$V_f = \frac{\pi D^2}{4}\left(C_{pf}\, 2.4D\right) \tag{3}$$

$$V_a = \frac{\pi D^2}{4}\left(C_{pa}\, 3.6D\right) \tag{4}$$

$$V_{PMB} = \frac{\pi D^2}{4}\left(L - 6D\right) \tag{5}$$

The above can be combined into the following,

$$V = \frac{\pi D^3}{4}\left[3.6C_{pa} + \frac{L}{D} - 6 + 2.4C_{pf}\right] \tag{6}$$

It can be easily shown that the prismatic coefficients can be calculated as,

$$C_{pf} = \int_0^1 (1 - x^{n_f})^{2/n_f}\, dx \tag{7}$$

$$C_{pa} = \int_0^1 (1 - x^{n_a})^2\, dx \tag{8}$$

These integrals are numerically evaluated using the built in “quad” function in Matlab, although analytic evaluation is possible,

$$C_{pa} = \frac{2n_a^2}{1+3n_a+2n_a^2} \quad \textbf{(9)}$$

$$C_{pf} = \frac{\Gamma\left(1-\frac{2}{n_f}\right)\Gamma\left(\frac{1}{n_f}\right)}{n_f\,\Gamma\left(1+\frac{33}{n_f}\right)} \quad \textbf{(10)}$$

where the Gamma function is defined in Ref. 4 (Abramowitz and Stegun, 1970).

Initially, the total volume is calculated for the values of L =360 ft., D = 30 ft., n_a = 3.0 and n_f = 3.0, by using Equations (6) through (8), and yielded a value of 217337.73 ft^3. This value is kept constant throughout the calculations.

For the limited length case, L is kept constant at 360 ft. The shape factors n_a and n_f are varied between 2.0, 3.0 and 4.0. The prismatic coefficients C_{pa} and C_{pf} are calculated for each of them by evaluating the integrals in Equations (7) and (8) numerically. Then the corresponding diameters are found by solving for the maximum hull diameter D in Equation (6). This is achieved by solving the following cubic equation,

$$\left(3.6C_{pa}+2.4C_{pf}-6\right)D^3+LD^2-\frac{4V}{\pi}=0 \quad \textbf{(11)}$$

Two iterations ensure that the solution converges to the value which meets the requirement to have L_a=3.6D and L_f=2.4D.

For the limited diameter case, D is kept constant at 30 ft. C_{pa} and C_{pf} are calculated the same way as the limited length case. Then the corresponding lengths are found by solving for L in Equation (6).The resulting equation becomes,

$$L = D\left(\frac{4V}{\pi}D^3-3.6C_{pa}-2.4C_{pf}+6\right) \quad \textbf{(12)}$$

Two Matlab programs (Appendix) are used to perform the calculations for each case. In the limited length case, the program “limlen” starts by inputting n_a and n_f, and computes the corresponding diameter. Similarly, for the limited diameter case, the program “limdia” computes the length for the given n_a and n_f values. These values are used as an input in the strip theory seakeeping prediction program.

B. MOTIONS IN A SEAWAY

Wave patterns in an open sea are ever changing with time and space, in a manner that appears to defy analysis be it linear or second order Stokes [Ref 5]. Ambient waves on the surface of the sea are dispersive as well as random. Random refers to the character of the wave height distribution. In a continuous distribution, the sinusoidal waves have continuously distributed amplitude and phase so that in summation the variation of wave height with time is not systematic in any respect, but random. The generating mechanism is, predominantly, the effect upon the water surface of wind in the atmosphere. The practically useful data extractable from a random wave record $h(t)$ is its spectral density, $S(\omega)$. The random $h(t)$ record is processed in such a way to produce a curve of $S(\omega)$ versus wave frequency, ω. The spectral density is obtained from a wave height record taken over a time period for which the sea conditions are assumed to be unchanging, in an average sense (stationary). This corresponds to a certain sea state. The function $S(\omega,\theta)$ is called the spectral energy density or simply the energy spectrum. More specifically, this is a directional energy spectrum; it can be integrated over all wave directions to give the frequency spectrum

$$S(\omega) = \int_0^{2\pi} S(\omega,\theta) d\theta \quad . \tag{13}$$

Usually in the fields of ocean engineering and naval architecture it is customary to assume that the waves are long crested which means the fluid motion is two dimensional and the wave crests are parallel. With such a simplification it is possible to use existing information for the frequency spectrum (13), which is based on a combination of theory and full scale observations. The sea spectrum (spectral density) gives us information on mean wave height within finite frequency bands. Since most of the wave energy is within a relatively small range of wave lengths where it may resonate the ship, we can model the seaway as a narrow band random process.

For most purposes we are interested primarily in the larger waves. The most common parameter that takes this into account is the significant wave height, $H_{1/3}$, defined as the average of the highest one third of all waves. This is computed by

$$H_{1/3} = 4.0(m_0)^{1/2} . \tag{14}$$

In this equation, m_0 is the area under the spectrum $S(\omega)$ integrated over the entire range of frequencies ω. An average frequency of the spectrum can be defined as the expected number of zero upcrossings per unit time, that is, the number of times the wave amplitude passes through zero with positive slope. The final result here is

$$\omega_z = \left(\frac{m_2}{m_0} \right)^{1/2} . \tag{15}$$

The average period between zero upcrossings is

$$T_z = \frac{2\pi}{\omega_z} = 2\pi \sqrt{\frac{m_0}{m_2}} \tag{16}$$

More meaningful frequency parameters can be obtained from the set of moments, which depend on spectrum shape

$$m_n = \int_0^\infty \omega^n S(\omega) d\omega \ , \qquad n=0,1,2,... \qquad \textbf{(17)}$$

In particular, the area, m_0, is the variance or the total energy of the spectrum. Also m_2 is variance of velocity and m_4 is variance of acceleration.

A good model for fully developed seas is the classical Pierson-Moskowitz spectrum. This spectral form depends upon a single parameter which is the significant wave height. It is intended to represent point spectrum of a fully-developed sea. Fetch and duration are assumed to sufficiently large so that the sea has reached steady state, in a statistical sense. This spectral family should be recognized as an asymptotic form, reached after an extended period of steady wind, with no contamination from an underlying swell. Using the spectral family, along with the similarity theory of S. A. Kitaigorodskii, Pierson and Moskowitz (1964) [Ref. 5] arrived at the following analytical formulation for ideal sea spectra,

$$S_1^+(\omega) = \frac{0.0081 g^2}{\omega^5} \exp\left[-0.032 \left(\frac{g}{H_{1/3}\omega^2} \right)^2 \right] , \qquad \textbf{(18)}$$

where

$S_1^+(\omega)$ = one-sided incident wave spectrum

g = acceleration of gravity

$H_{1/3}$ = significant wave height

ω = wave frequency

In Figure 4 we can observe typical Pierson-Moskowitz wave spectra for 5 m. significant wave height.

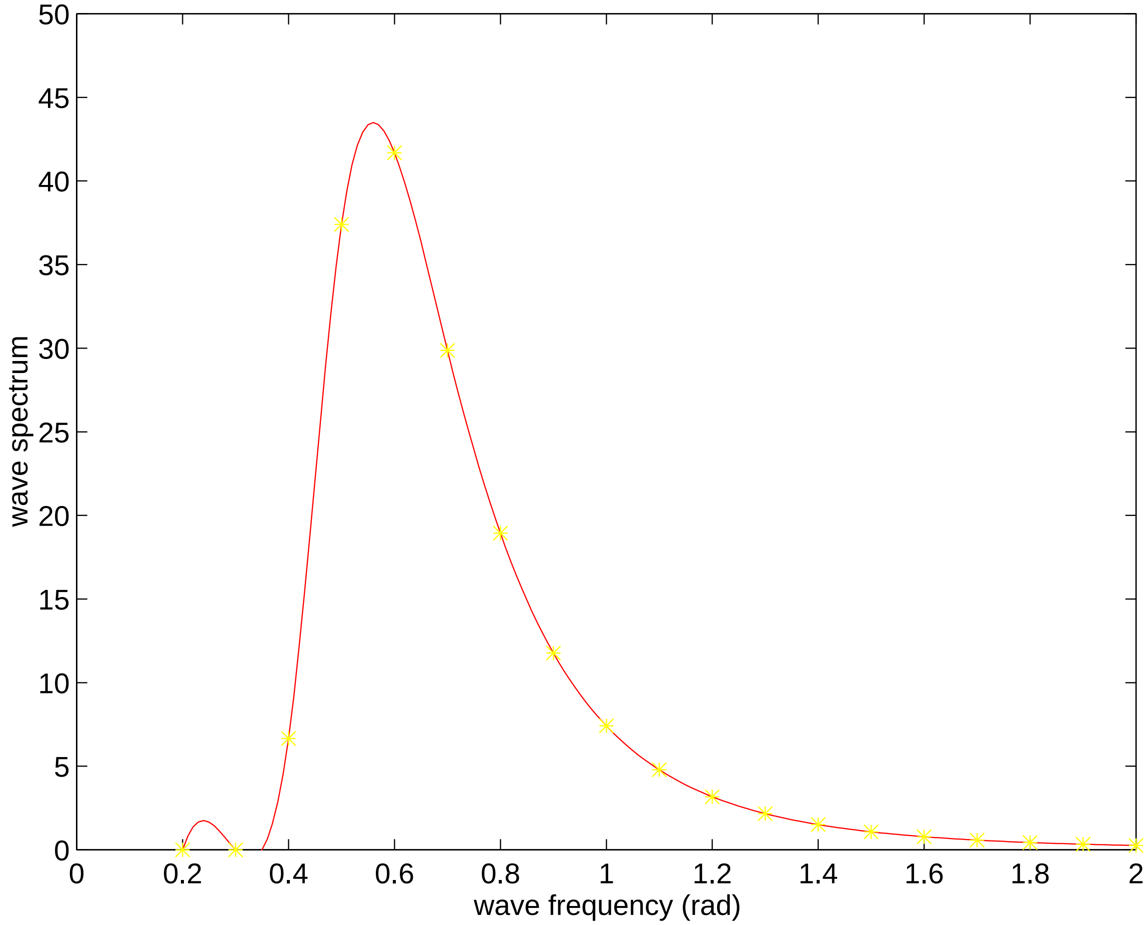

Figure 4. Typical Pierson-Moskowitz wave spectra

Any conclusions drawn on the seakeeping behavior of a ship based on the critical examination of motion response in regular waves can, at best, assume only academic significance. The establishment of the seakeeping behavior of a ship has to be done in a realistic seaway. With the spectral description of sea waves given before, we can return to the subject of body motions and generalize the results of regular harmonic waves. If the sea waves are described by the random distribution , and if the response of the body to each component wave is defined by a response amplitude operator $Z(\omega,\theta)$, the body response will be

$$\eta_j(t) = \Re \iint Z_j(\omega,\theta) e^{i\omega t} dA(\omega,\theta) \ . \qquad \textbf{(19)}$$

The principal assumption here is that linear superposition applies, as it must in any event for the underlying development of the RAO and the spectrum.

Like the waves themselves, the response (19) is a random variable. The statistic of the body response are identical to the wave statistics, except that the wave energy spectrum S is multiplied by the square of the RAO (this is a property of linear systems). Thus, if the subscript R represents any body response, we have

$$S_R(\omega) = \left| Z_R(\omega) \right|^2 S(\omega) \ , \qquad \textbf{(20)}$$

where $Z_R(\omega)$ is the RAO of the response R, and $S(\omega)$ the spectrum of the seaway. Equation (20) can then be utilized to obtain the spectrum of the response R. Figure 5 displays the spectrum of response of the relative vertical motion at the top of our model submarine's sail while submarine's forward speed is 5 Knots and it is at 3 submarine diameter depth. Also seaway is modeled by Pierson-Moskowitz spectrum with 5 m. significant wave height and head seas.

To a large extent, equation (20) provides the justification for studying regular wave responses. The transfer function $Z_R(\omega)$ is valid not only in regular waves, where it has been derived, but also in a superposition of regular waves, and ultimately in a spectrum of random waves. Generally speaking, a vessel with favorable response characteristics in regular waves will be good in irregular waves, and vice versa.

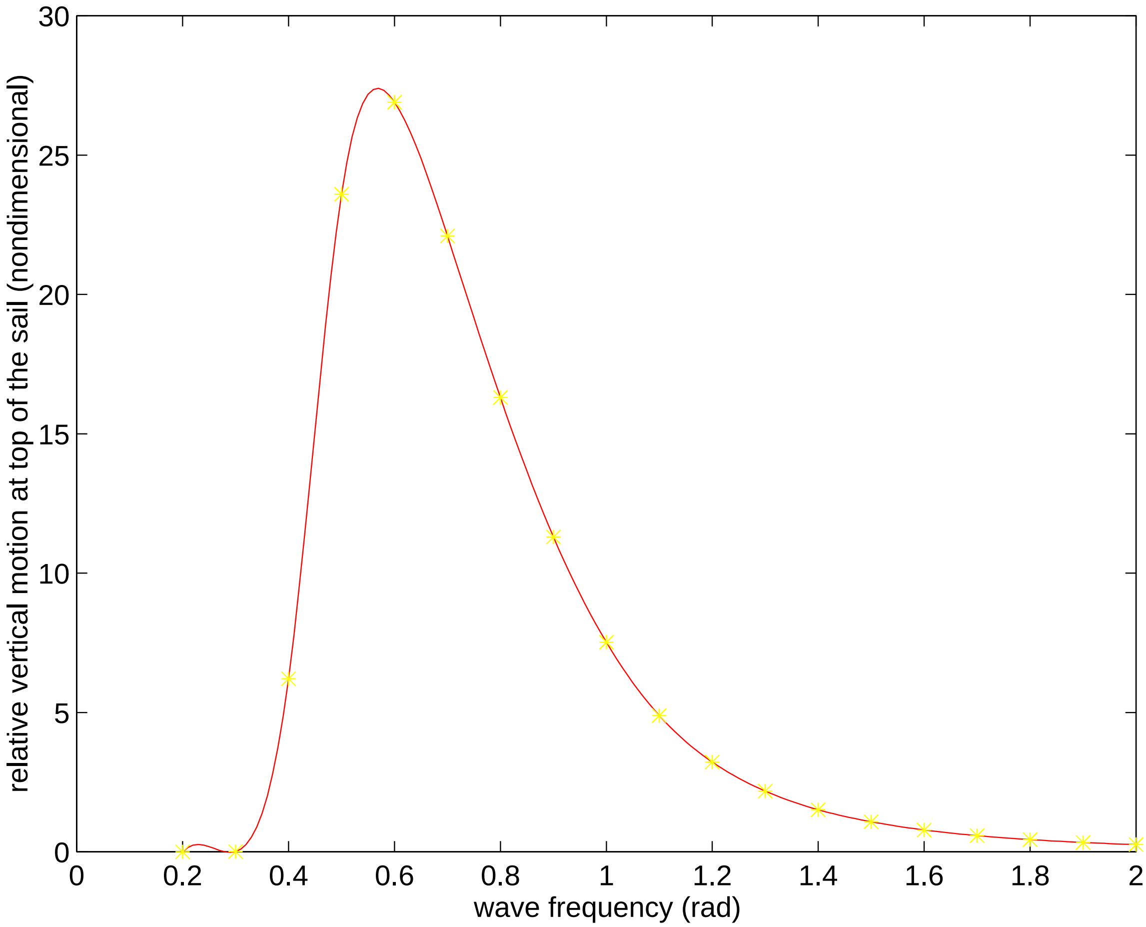

Figure 5. Spectrum of response for relative vertical motion

The average period between zero upcrossings was determined by Equation (16), and the number between zero upcrossings per unit time is

$$N_z^R = \frac{1}{2\pi}\sqrt{\frac{m_2^R}{m_0^R}} \ , \tag{21}$$

where m_0^R, m_2^R are the moments of the particular response R, whose spectral density is given by Equation (20). Equation (21) can be generalized for the case of the average number of upcrossings above a specified level α as in

$$N_{z,\alpha}^R = \frac{1}{2\pi}\sqrt{\frac{m_2^R}{m_0^R}} \exp\left(-\frac{\alpha^2}{2m_0^R}\right). \tag{22}$$

Equation (22) can be utilized to determine such events deck wetness and bow slamming for a surface ship or periscope submergence and sail broaching for a near surface submarine. If f represents height of the periscope over calm sea surface level, the number of periscope submergence events per hour is

$$N_p = 3600 \frac{1}{2\pi} \sqrt{\frac{m_2}{m_0}} \exp\left(-\frac{f^2}{2m_0} \right) , \tag{23}$$

where m_0 , m_2 are the moments of the vertical relative motion spectrum at periscope. The same equation can be used to estimate the frequency of sail broaching, with f substituted by the distance between top of the sail and encountered wave surface. Of course, m_0 , m_2 are now the moments of the relative motion spectrum at top of the sail.